

VI

Deductive Arguments

The arguments we have Considered so far are all uncertain in one way or another. New examples might always refute an argument from example, and even an informed and impartial source might be wrong. Properly formed *deductive arguments*, however, are arguments the truth whose premises guarantees the truth of their conclusions.

If there are no chance factors in chess, then chess is a game of pure skill.
There are no chance factors in chess.
Therefore, chess is a game of pure skill.

If these two premises are true, then it *must* also be true that chess is a game of pure skill. To disagree with conclusion, you'd also have to disagree with at least one of the premises.

Deductive arguments offer certainty, then –but only if their *premises* are also certain. Since the premises of our arguments in fact are seldom certain, the conclusions of real-life deductive arguments still have to be taken with a few (sometimes many!) grains of salt. Still, when strong premises can be found, deductive are very useful. Remember rule 3: try to start with reliable premises.

Even when the premises are uncertain, deductive forms offer an effective way of *organizing* an argument, especially an argumentative essay. This chapter presents six common deductive forms with simple examples, each in a section of its own. Chapters VII-IX return to their use in argumentative.

(24) Modus Ponens

Properly formed deductive arguments are called *valid* arguments. Using the letters **p** and **q** to stand for sentences, the simplest valid deductive form is:

If [sentence **p**] then [sentence **q**].
[Sentence **p**].
Therefore, [sentence **q**].

Or, more briefly:

If **p** then **q**.
p
Therefore, **q**.

This form is called “modus ponens” (“the mode of putting”: put **p** get **q**). Taking **p** to stand for “There are no chance factors in chess” and **q** to stand for “Chess is a game of pure skill,” our introductory example follows modus ponens. Check it out.

Often an argument in this form is so obvious that it does not need to be stated as an official modus ponens at all.

Since optimists are more likely to succeed than pessimists, you should be an optimist.

This argument could be written:

If optimists are more likely to succeed than pessimists, then you should be an optimist.

Optimists *are* more likely to succeed than pessimists.

Therefore, you should be an optimist.

but the argument is perfectly clear without putting it in this form. At other times, however, writing out the modus ponens is useful:

If there are millions of habitable planets in our galaxy, then it seems likely that life has evolved on more than just this one.

There are millions of habitable planets in our galaxy.

Therefore it seems likely that life has evolved on more than just this one.

To develop this argument you must explain and defend both of its premises, and they require quite different arguments (Why?). It is useful to state them clearly and separately from the start.

(25) Modus Tollens

A second valid form is “modus tollens” (“the mode of taking”: take **q**, take **p**):

If **p** then **q**.

Not-**q**

Therefore, not-**p**.

Here “Not-**q**” simply stands for the denial of **q**, i.e., for the sentence “It is not true that **q**”; similarly for “not-**p**.”

Remember Sherlock Holmes’s argument, which we discussed in section 1:

A dog was kept in the stalls, and yet, though someone had been in and fetched out a horse, he had not barked... Obviously the visitor was someone whom the dog knew well...

Holmes’s argument is a modus tollens:

If the dog did not know the visitor well, then the dog would have barked.

The dog did not bark.

Therefore, the dog knew the visitor well.

To write this argument in symbols, use **k** for “The dog did not know the visitor well” and **b** for “The dog barked.”

If **k** then **b**.

Not-**b**

Therefore, not-**k**.

“Not-**b**” stands for “The dog did not bark,” and “not-**k**” stands for “It is not true that the dog did not know the visitor well,” i.e. “The dog *did* know the visitor well.”¹³

Astronomer Fred Hoyle wields an interesting modus tollens. To paraphrase a bit:

If the universe were infinitely old, there would be no hydrogen left in it, since hydrogen is steadily converted into helium throughout the universe, and this conversion is a one-way process. But in fact the universe consists almost entirely of hydrogen. Thus the universe must have had a definite beginning.

To put Hoyle’s argument in symbols, use **i** to stand for “The universe is infinitely old” and **h** to stand for “No hydrogen is left in the universe.”

If **i** then **h**.

Not-**h**

Therefore, not-**i**.

“Not-**h**” stands for “It is not true that there is no hydrogen left in the universe” (or: “The universe does contain hydrogen”); “not-**i**” means “It is not true that the universe is infinitely old.” Hoyle goes on to rephrase the conclusion: because the universe is not infinitely old, there must have been a definite point at which it began.

(26) Hypothetical Syllogism

A third valid form is “hypothetical syllogism”:

If **p** then **q**.

If **q** then **r**.

Therefore, if **p** then **r**.

For instance:

If you study other cultures, then you realize what a variety of human customs there is.

¹³ We might instead have defined **k** as “The dog did know the visitor well,” so that in symbols the argument goes:

If not-**k** then **b**.

Not-**b**

Therefore, **k**.

Strictly speaking, the conclusion then is “Not-not-**k**” – “It is not true that it is not true that the dog knew the visitor well” –but this is equivalent simply to **k**.

If you **realized** what a variety of human customs there is, then you **question** your own customs.

Therefore, if you **study** other cultures, then you **question** your own customs.

Using the letters in boldface to stand for the component sentences in this statement, we have:

If **s** then **r**.

If **r** then **q**.

Therefore, if **s** then **q**.

Hypothetical syllogism is valid for any number of premises as long as each premise has the form “If **p** then **q**” and the **q** of one premise becomes the **p** of the next. In section 5, for example we considered an argument with two premises above also a third:

If you **question** your own customs, then you become more **tolerant**.

From this and the two premises above, you can validly conclude “If **s** then **t**” by hypothetical syllogism.

Notice that hypothetical syllogism offers a good model for explaining the connections between cause and effect (rule 18). The conclusion links a cause and an effect, while the premises explain the stages in between.

(27) Disjunctive Syllogism

A fourth valid form is “disjunctive syllogism”:

p or **q**.

not-**p**.

Therefore, **q**.

Consider, for instance, Bertrand Russell’s argument discussed in section 2:

Either we hope for progress through improving **morals**, or we hope for progress through improved **intelligence**.

We can’t hope for progress through improving **morals**.

Therefore, we must hope for progress through improving **intelligence**.

Again, using the boldface letters as symbols, this argument goes:

m or **i**.

not-**m**.

Therefore, **i**.

In English the word “or” can have different meanings. In its “exclusive” sense, the word “or” in the sentence “**a** or **b**” means either **a** or **b** is true, but both. In its “inclusive” sense “**a** or **b**”

means that either **a** or **b** is true, and possibly both. Disjunctive syllogisms are valid regardless of which sense of “or” is used. In the exclusive sense of “or,” one can also argue:

p or **q**.

p.

Therefore, not-**q**.

Again this form is *only* valid in the exclusive sense of “or.” Suppose, for example, that someone argues:

Only Zbigniew or Zoltan could have done the dreadful deed. Zoltan did it; therefore Zbigniew did not.

Zbigniew’s innocence depends on what the word “or” means in the first sentence. If the dreadful deed was something that only one person could have done, then the “or” is exclusive and the argument is valid. If the dreadful deed could have been a joint production, however, then the “or” remains inclusive, and Zoltan’s guilt does not prove Zbigniew’s innocence.

(28) Dilemma

A fifth valid form is the “dilemma.”

p or **q**.

If **p** then **r**.

If **q** then **s**.

Therefore, **r** or **s**.

Here is an example from Edmund Burke:

There is a dilemma to which every opposition to successful iniquity must, in the nature of things, be liable. If you lie still, you are considered an accomplice to the measures to which you silently acquiesce. If you resist, you will be accused of provoking irritable power to new excesses. The conduct of a losing party never appears right...

Let us translate the argument into somewhat clear and more consistent terms as we also translate it into dilemma form. Use **r** to stand for “You resist successful iniquity”; **a** for “You are considered an accomplice”; and **p** for “You are accused of causing new provocations.” In symbols, then, the argument could be put:

r or not-**r**.

If **r** then **p**.

If not-**r** then **a**.

Therefore, **p** or **a**.

Notice that we did not need a separate symbol for the sentence “You lie still”: we translated it as “not-**r**.” Notice also that the premises are somewhat rearranged, and the first premise made more explicit, to fit the dilemma form.

Burke concludes that “the conduct of a losing party never appears right.” In a way this is only a restatement of the conclusion we have put as “**p** or **a**.” We could also make it more explicit by spelling out another argument in dilemma form, beginning with the conclusion of the last one:

Either you are accused of causing new **p**rovocations or you are considered an **a**ccomplice.
 If you are accused of causing new **p**rovocations, your conduct appears **w**rong.
 If you are considered an **a**ccomplice, your conduct appears **w**rong.
 Therefore, your conduct always appears **w**rong.

In symbols:

Either **p** or **a**.
 If **p** then **w**.
 If **a** then **w**.
 Therefore, **w**.

Technically the conclusion is **w** or **w**,” but we don’t really need to say it twice.

(29) Reductio ad absurdum

One traditional deductive strategy deserves special mention even though, strictly speaking, it is only a version of modus tollens. This is “reductio ad absurdum” i.e., a “reduction to absurdity.”

To prove: **p**.
Assume: Not-**p** (that is, that **p** is false.)
From the assumption derive an implication: **q**.
Show: **q** is false (contradictory, silly, “absurd”).
Conclude: **p**.

Arguments by “reductio,” as these arguments are often called, thus establish their conclusions by showing that the denial of the conclusion leads to absurdity. There is nothing left to do, the argument suggests, but to accept the conclusion.

Remember, for example, the argument for the existence of a Creator which we discussed in section 12. Houses have creators, the argument goes, and the world is *like* a house –it too is ordered and beautiful. Thus, the analogy suggests, the world must have a Creator too. Section 12 quoted David Hume to the effect that the world is not relevantly enough similar to a house for this analogy to succeed. In Part V of his *Dialogues* Hume also suggests a reductio ad absurdum of the analogy. Paraphrasing:

Suppose that the world has a creator like a house does. Now when houses are not perfect, we know who to blame: the carpenters and masons who created them. But the *world* is also not wholly perfect. Therefore, it would seem to follow that the creator of the world, God is not perfect either. But you would consider this conclusion absurd. The only way to avoid the absurdity, however is to reject the supposition that leads to it. Therefore, the world does not have a Creator in the way that a house does.

Spelled out in “reductio” form, we have:

To prove: The world does not have a Creator in the way that a house does.

Assume: The world *does* have a Creator in the way a house does.

From the assumption it follows that: God is imperfect (because the world is imperfect).

But: God cannot be imperfect.

Therefore: The world does not have a Creator in the way a house does.¹⁴

Not everyone would find the idea of an imperfect God “absurd,” of course, but Hume knew that the Christians with whom he was arguing would not accept it.

(30) Deductive arguments in several steps

Many valid forms are *combinations* of the simple forms introduced in sections 24–29. Here, for example, is Sherlock Holmes performing a simple deduction for Doctor Watson’s edification, meanwhile commenting on the relative roles of observation and deduction. Holmes has casually remarked that Watson has been to a certain Post Office that morning, and furthermore that he sent off a telegram while there. “Right!” replies Watson, amazed, “right on both points! But I confess that I don’t see how you arrived at it.” Holmes:

“It is simplicity itself... Observation tell me that you have a little reddish mold adhering to your instep. Just opposite the Wigmore Street Post Office they have taken up the pavement and thrown up some earth, which lies in such a way that it is difficult to avoid treading it in entering. The earth is of this peculiar reddish tint which is found, as far as I know, nowhere else in the neighborhood. So much is observation. The rest is deduction.”

[Watson]: “How, then did you deduce the telegram?”

[Holmes]: “Why, of course I knew that you had not written a letter, since I sat opposite you all morning. I see also in your open desk there that you have a sheet of stamps and a thick bundle of postcards. What could you go to the Post office for, then, but to send a wire? Eliminate all the other factors and the one which remains must be the truth.”¹⁵

Putting Holmes’s deduction into more explicit premises, we might have:

1. Watson has a little reddish mold on his boots.
2. If Watson has a little reddish mold on his boots, then he has been to the Wigmore Street Post Office this morning (because there and only there is reddish dirt of that sort thrown up, and in a way difficult to avoid stepping in).
3. If Watson has been to the Wigmore Street Post Office this morning, he either mailed a letter, bought stamps or cards, or sent a wire.
4. If Watson had gone to the Post Office to mail a letter, he would have written the letter this morning.
5. Watson wrote no letter this morning.
6. If Watson had to the Post Office this morning to buy stamps or cards, he would not already have a drawer full of stamps and cards.

¹⁴ For practice, translate this argument as a modus tollens.

¹⁵ A. Conan Doyle, *The Sign of the Four*, Chapter I.

7. Watson already has a drawer full of stamps and cards.
8. Therefore, Watson sent a wire at the Wigmore Street Post Office this morning.

We need to break the argument down into a series of valid arguments in the simple forms presented in sections 24-29. We might start with a hypothetical syllogism:

2. If Watson has a little reddish mold on his boots, then he has been to the Wigmore Street Post Office this morning.
3. If Watson has been to the Wigmore Street Post Office this morning, he either mailed a letter, bought stamps or cards, or sent a wire.
- A. Therefore, if Watson has a little reddish mold on his boots, he either mailed a letter, bought stamps or cards, or sent a wire at the Wigmore Street Post Office this morning.

(I will use “A,” or “B,” etc. to stand for the conclusions of the simple arguments, which can then be used as premises to draw further conclusions.) Now with A and 1 we can use modus ponens:

- A. If Watson has a little reddish mold on his boots, he either mailed a letter, bought stamps or cards, or sent a wire at the Wigmore Street Post Office this morning.
1. Watson has a little reddish mold on his boots.
- B. Therefore, Watson either mailed a letter, bought stamps or cards, or sent a wire at the Wigmore Street Post Office this morning.

Two of these possibilities can now be ruled out, both by modus tollens.

4. If Watson had gone to the Post Office to mail a letter, he would have written the letter this morning.
5. Watson wrote no letter this morning.
- C. Therefore, Watson did not go to the Post Office to mail a letter.

and:

6. If Watson had to the Post Office this morning to buy stamps or cards, he would not already have a drawer full of stamps and cards.
7. Watson already has a drawer full of stamps and cards.
- D. Therefore, Watson did not go the Post Office to buy stamps or cards.

Finally, then:

- B. Watson either mailed a letter, bought stamps or cards, or sent a wire at the Wigmore Street Post Office this morning.
- C. Watson did not go to the Post Office to mail a letter.
- D. Watson did not go the Post Office to buy stamps or cards.
8. Therefore, Watson sent a wire at the Wigmore Street Post Office this morning.

This last inference is an extended disjunctive syllogism. “Eliminate all the other factors, and the one which remains must be the truth.”